Mathematical Modelling

In our daily life situations and real world we come across many problems, for which solutions are found by mathematicians, scientists, economists, psychologists and so on.

Study the following examples.

* Finding the height of unreachable mountains.
* Finding the depth of rivers and oceans.
* Predicting the population of a city or a country after 10 years.
* Estimating the temperature on the surface of sun.
* Launching a satellite to the mars.
* Creating a communication network through satellite

All such real world problems are solved by using a concept known as **mathematical modelling.**

What is mathematical modelling?

Mathematical model is a mathematical description of some real life situation. It is a process of creating a mathematical model of a problem and using it to analyse and solve the problem.

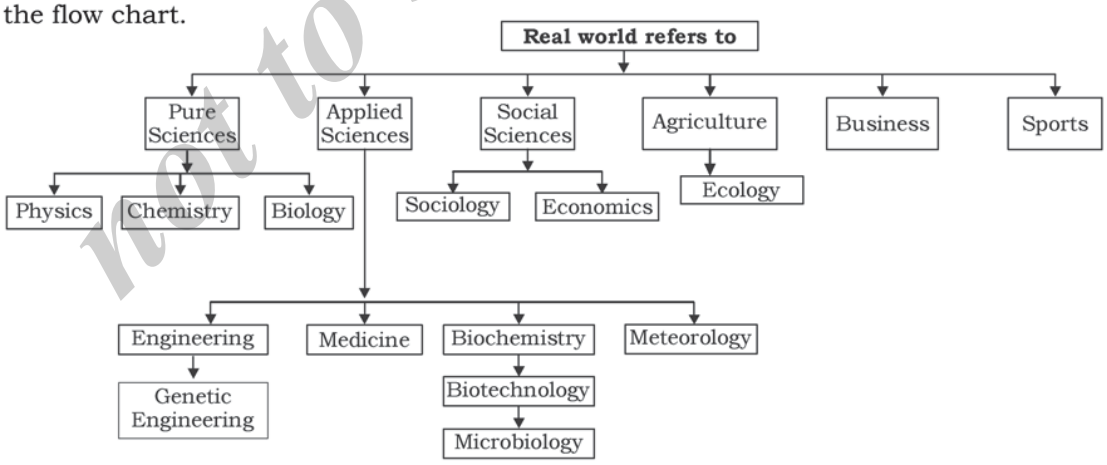
A mathematical model is a description of a system using mathematical concepts and language.

The process of developing a mathematical model is termed as mathematical modelling.

Mathematical modelling is the use of mathematics to,

* describe real world phenomena
* investigate important questions about real world situations.
* expain real world phenomena in mathematical form.
* test the ideas in mathematical ways.
* make predictions about the real world.

The real world represented by mathematical models refers to the areas given in the flow chart.



Mathematical models are used in the sub – areas of each of the above mentioned areas and almost all branches of knowledge.

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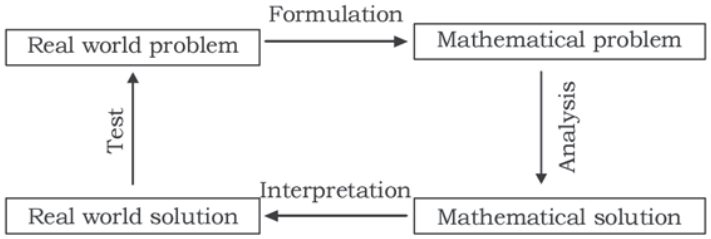
The process of mathematical modelling is represented in the diagram given below.



The process of mathematical modelling is carried on following certain stages in sequential order.

The stages of mathematical modelling

Stage1: Understanding the problem

In this stage, the real world problem is defined and stated in terms of certain factors and assumptions which can be practically understood.

For example, to estimate the number of trees in a reserve forest, it is not possible to count each and every tree.

Usually, a sample area of the forest is considered and total number of trees in the forest is estimated.

Stage 2: Formulation of mathematical model.

The mathematical terms, factors and assumptions are selected to establish a relationship representing the real world problem. Mathematical model is developed and described in mathematical terms and language. Some ways to describe the mathematical model are,

* Define variables
* Gather data and organise into tables
* Form equations and formulae
* Draw graphs

For example, in order to estimate the total number of trees in the forest, a mathematical model has to be developed.

Suppose the forest area is 100 km^2. It may be divided into three types of areas.

A – area which is thickly covered with trees

B – area which is sparcely covered which trees

C – area which is neither thickly nor sparcely covered with trees.

Assume that the entire forest area is divided into 200 parts of half km^2 each. This can be represented by drawing a map using scale drawing. Out of 200 parts, the number of each type is found out and the ratio is A: B: C = 3 : 1 : 2

Let us suppose 12 parts are randomly selected, where 6 are of type A, 2 are type B and 4 are type C. The trees in these 12 parts are numbered and counted.

So, the data is collected for analysis.

Step 3: Solving the mathematical problem

The simplified mathematical model developed in step 2 is solved in this step using various mathematical techniques.

For example, after counting the number of trees in each of the selected 12 parts, average of the number of trees is calculated.

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Suppose in 12 parts, average number of trees is 3556.

Then, average number of trees in one part is $\frac{3556}{12} = 296$

$\therefore$ the total number of trees in the 200 parts (100 km^2 area) = 200 × 296 = 59,200

Step 4: Interpreting the solution

The solution obtained in step 3 is now interpreted in the context of the real life situation or real world problem. That is, explanations are given and predictions made about the solution of the problem.

For example, the problem in step 1 is revisited and solution is applied.

i.e., the total number of trees in 100 km^2 area of the forest is approximately equal to 59,200.

Step 5: Validating the model

The results obtained in step 3 and interpreted in step 4 is checked to see if it makes sense. It so, the model is used until assumptions change or new information becomes available or new mathematical model is evolved.

Alternate model:

Suppose an aerial photograph the forest is taken and each tree is represented by a dot in the picture.

Develop a model to find the number of trees.

Now let us consider another example from commercial mathematics leading to mathematical modelling.

Example: Rajesh wants to buy a bicycle. He goes to the shop and selects a bicycle whose cost is ₹1800. He has ₹600 with him. The shopkeeper offers an instalment buying scheme as follows:

Rajesh can make a cash down payment of ₹600 and the remaining money could be paid in two monthly instalments of ₹610 each. Rajesh has two options, one is to go for instalment buying scheme or make full cash payment by taking loan from a bank which is available at the rate of 10% per annum simple interest. Which option is more economical to him?

Solution:

Step 1 (understanding the problem) :

Rajesh has two offers. He needs to determine whether he take the offer made by the shopkeeper or not. For this, he should know the two rates of interest the one changed by the shopkeeper in the instalment scheme and the other by the bank. Compare them and decide which is more economical.

Step 2 (Formulation of mathematical model) :

Observe that the entire money paid in the instalment scheme is within a year and hence simple interest shall be charges.

Cash price of the bicycle = ₹1800

Cash down payment under instalment scheme = ₹600

$\therefore$ the balance price the needs to be paid in the instalment scheme = ₹(1800-600)=1200.

Let r% p.a. be the rate of interest charged by the shopkeeper.

Amount of each instalment = ₹610

Total amount paid in instalments = ₹610 + ₹610 = ₹1220

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$\therefore$ Interest paid in instalment scheme = ₹1220 - ₹1200
= ₹20.....(i)

Principal for the first month = ₹1200

Principal for the second month = ₹(1200–610) = ₹590

$\therefore$ Second instalment = Second principal + interest charged
= ₹590 + ₹20 = ₹610

$\therefore$ the total principal for one month = ₹1200 + ₹590
= ₹1790

We know, Interest = $\frac{PTR}{100}$

$\therefore \quad I = \frac{1790 \times r \times 1}{100 \times 12}$ ..... (ii)

Step 3 (Solving the problem) :

From (i) and (ii) we get,

$\frac{1790 \times r \times 1}{100 \times 12} = 20$

$\therefore \quad r = \frac{20 \times 1200}{1790} = 13.14$

Step 4 (Interpreting the solution) :

The rate of interest charged in the instalment scheme = 13.14%

The rate of interest charged by the bank = 10%

So, Rajesh should prefer to borrow the money from the bank to buy the bicycle which is more economical.

Step 5 (Validating the model)

In this example, validating the model is not of much importance as the values are fixed.

However, if the process of taking loan is different or the bank charges compound interest, then comparison of rate of interest may be different and Rajesh may change the decision.

Importance of mathematical modelling

Mathematical modelling is an interdisciplinary subject. Mathematicians and specialists in other fields share their knowledge and expertise to improve the existing results, solutions, products and develop better ones.

The importance of mathematical modelling are

* to gain understanding of the real world systems
* to predict or forecast the results
* to estimate the results when the data is enormous.

Note: Collect problems related to profit and loss, discount, instalment buying, hire purchase, compound interest and banking. Develop a mathematical model to solve them.

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